Peer-to-Peer Systems

Analysis of unstructured P2P systems

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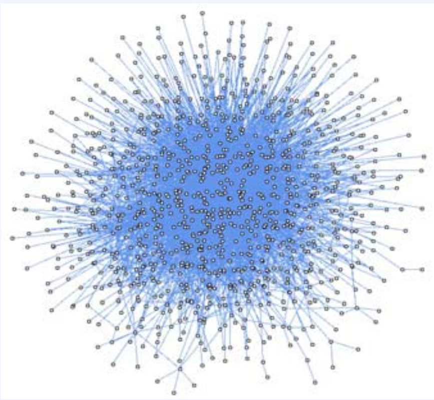
Some questions…

- How scalable is Gnutella?
- How robust is Gnutella?
- Why does FreeNet work?
- What would an ideal (unstructured) P2P system look like?
- What is do the overlay networks of existing (unstructured) P2P systems look like?



Gnutella snapshot, 2000

Scalability of Gnutella: quick answer

- Bandwidth Generated in Bytes (Message 83 bytes)
  - Searching for a 18 byte string

	T=2	T=3	T=4	T=5	T=6	T=7	T=8
N=2	332	498	664	830	996	1,162	1,328
N=3	747	1,743	3,735	7,719	15,687	31,623	63,495
N=4	1,328	4,316	13,280	40,172	120,848	362,876	1,088,960
N=5	2,075	8,715	35,275	141,515	566,475	2,266,315	9,065,675
N=6	2,988	15,438	77,688	388,938	1,945,188	9,726,438	48,632,688
N=7	4,067	24,983	150,479	903,455	5,421,311	35,528,447	192,171,263
N=8	5,312	37,848	262,600	1,859,864	13,019,712	91,138,648	637,971,200

  - N = number of connections open
  - T = number of hops

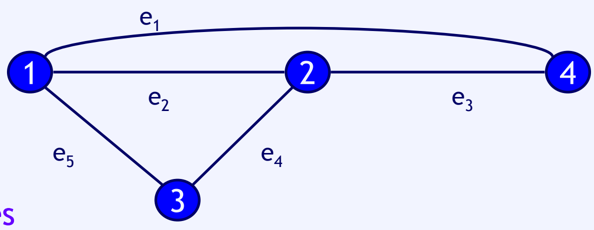
Source: Jordan Ritter: Why Gnutella Can't Scale. No, Really.

Graphs

- Rigorous analysis of P2P systems: based on graph theory
  - Refresher of graph theory needed
- First: graph families and models
  - Random graphs
  - Small world graphs
  - Scale-free graphs
- Then: graph theory and P2P
  - How are the graph properties reflected in real systems?
    - Users (peers) are represented by vertices in the graph
    - Edges represent connections in the overlay (routing table entries)
- Concept of self-organization
  - Network structures emerge from simple rules
  - E.g. also in social networks, www, actors playing together in movies

What Is a Graph?

- Definition of a graph:
  Graph $G = (V, E)$ consists of two finite sets, set V of vertices (nodes) and set E of edges (arcs) for which the following applies:
  1. If $e \in E$, then exists $(v, u) \in V \times V$, such that $v \in e$ and $u \in e$
  2. If $e \in E$ and above (v, u) exists, and further for $(x, y) \in V \times V$ applies $x \in e$ and $y \in e$, then $\{v, u\} = \{x, y\}$



Example graph with 4 vertices and 5 edges

Properties of Graphs

- An edge $e \in E$ is directed if the start and end vertices in condition 2 above are identical: $v = x$ and $y = u$
- An edge $e \in E$ is undirected if $v = x$ and $y = u$ as well as $v = y$ and $u = x$ are possible
- A graph G is directed (undirected) if the above property holds for all edges
- A *loop* is an edge with identical endpoints
- Graph $G_1 = (V_1, E_1)$ is a subgraph of $G = (V, E)$, if $V_1 \subseteq V$ and $E_1 \subseteq E$ (such that conditions 1 and 2 are met)

Important Types of Graphs

- Vertices $v, u \in V$ are connected if there is a path from v to u: $(v, v_2), (v_2, v_3), \ldots, (v_{k-1}, u) \in E$
- Graph G is connected if all $v, u \in V$ are connected
- Undirected, connected, acyclic graph is called a tree
  - Sidenote: Undirected, acyclic graph which is not connected is called a forest
- Directed, connected, acyclic graph is also called DAG
  - DAG = Directed Acyclic Graph (connected is assumed)
- An induced graph $G(V_C) = (V_C, E_C)$ is a graph $V_C \subseteq V$ and with edges $E_C = \{e = (i, j) \mid i, j \in V_C\}$
- An induced graph is a component if it is connected

Vertex Degree

- In graph $G = (V, E)$, the degree of vertex $v \in V$ is the total number of edges $(v, u) \in E$ and $(u, v) \in E$
  - Degree is the number of edges which touch a vertex
- For directed graph, we distinguish between in-degree and out-degree
  - In-degree is number of edges coming to a vertex
  - Out-degree is number of edges going away from a vertex
- The degree of a vertex can be obtained as:
  - Sum of the elements in its row in the incidence matrix
  - Length of its vertex incidence list

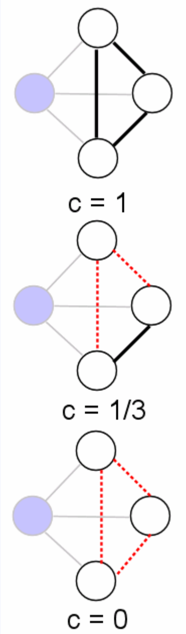
Important Graph Metrics

- Distance: $d(v, u)$ between vertices v and u is the length of the shortest path between v and u
- Average path length: Sum of the distances over all pairs of nodes divided by the number of pairs
- Diameter: $d(G)$ of graph G is the maximum of $d(v, u)$ for all $v, u \in V$
- Order: the number of vertices in a graph
- Clustering coefficient: number of edges between neighbors divided by maximum number of edges between them

$$C(i) = \frac{E(N(i))}{d(i)(d(i)-1)}$$

$E(N(i))$ = number of edges between neighbors of i

$d(i)$ = degree of i



Source: Wikipedia

Random Graphs

- Random graphs are first widely studied graph family
  - Many P2P networks choose neighbors more or less randomly
- Two different notations generally used:
  - Erdös and Renyi
  - Gilbert (we will use this)
- Gilbert's definition: Graph $G_{n,p}$ (with n nodes) is a graph where the probability of an edge $e = (v, w)$ is p

Construction algorithm:

- For each possible edge, draw a random number
- If the number is smaller than p, then the edge exists
- p can be function of n or constant

Basic Results for Random Graphs

Giant Connected Component

Let $c > 0$ be a constant and $p = c/n$. If $c < 1$ every component of $G_{n,p}$ has order $O(\log N)$ with high probability. If $c > 1$ then there will be one component of size $n*(f(c) + O(1))$ where $f(c) > 0$, with high probability. All other components have size $O(\log N)$

- In plain English: Giant connected component emerges with high probability when average degree is about 1

Node degree distribution

- If we take a random node, how high is the probability *P(k)* that it has degree *k*?
- Node degree is Poisson distributed
  - Parameter c = expected number of occurrences

$$P(k) = \frac{c^k e^{-c}}{k!}$$

Clustering coefficient

- Clustering coefficient of a random graph is asymptotically equal to *p* with high probability

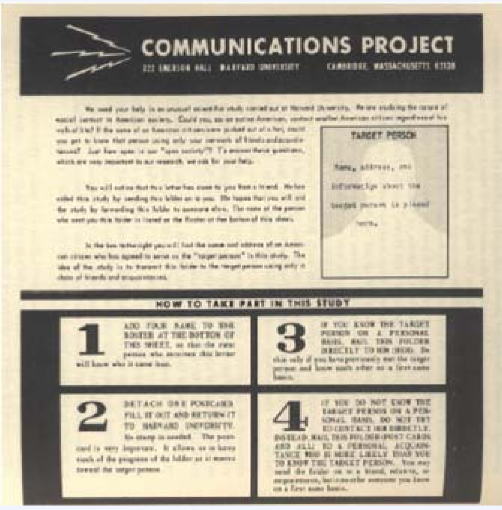
Random Graphs: Summary

- Before random graphs, regular graphs were popular
  - Regular: Every node has same degree
- Random graphs have two advantages over regular graphs
  1. Many interesting properties analytically solvable
  2. Much better for applications, e.g., social networks
- Note: Does not mean social networks are random graphs; just that the properties of social networks are well-described by random graphs
- Question: How to model networks with local clusters and small diameter?
- Answer: Small-world networks

Six Degrees of Separation

- Famous experiment from 1960’s (S. Milgram)
- Send a letter to random people in Kansas and Nebraska and ask people to forward letter to a person in Boston
  - Person identified by name, profession, and city
- Rule: Give letter only to people you know by first name and ask them to pass it on according to same rule
  - Some letters reached their goal
- Letter needed six steps on average to reach the person
- Graph theoretically: Social networks have dense local structure, but (apparently) small diameter
  - Generally referred to as “small world effect”
  - Usually, small number of persons act as “hubs”

Milgram's Small World Experiment


COMMUNICATIONS PROJECT

TARGET PERSON

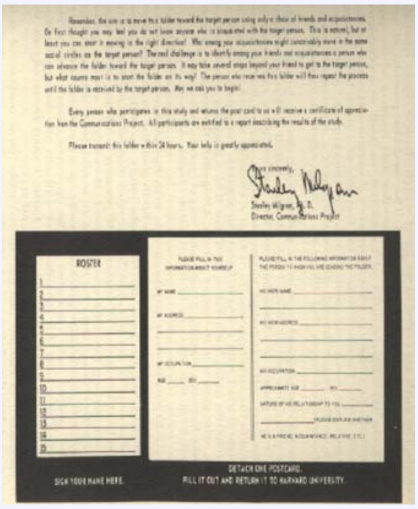
HOW TO TAKE PART IN THIS STUDY

1

2 DETACH ONE POSTCARD. FILL IT OUT AND RETURN IT TO HARVARD UNIVERSITY.

3 IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER).

4 IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY.


Please transmit this folder within 24 hours. Your help is greatly appreciated.

Stanley Milgram

Director, Communications Project

ROSTER

1.
2.
3.
4.
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8.
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10.
11.
12.
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14.
15.

SIGN YOUR NAME HERE.

DETACH ONE POSTCARD.
FILL IT OUT AND RETURN IT TO HARVARD UNIVERSITY.

Small-World Networks

- Developed/discovered by Watts and Strogatz (1998)
  - Over 30 years after Milgram's experiment!
- Watts and Strogatz looked at three networks
  - Film collaboration between actors, US power grid, Neural network of worm *C. elegans*
- Measured characteristics:
  - Clustering coefficient as a measure for 'regularity', or 'locality' of the network
    - If it is high, edges are rather build between neighbors than between far away nodes
  - The average path length between vertices
- Result
  - Most real-world networks have a **high clustering coefficient** (0.3-0.4), but nevertheless a **low average path length**
- Grid-like networks:
  - High clustering coefficient ⇒ high average path length (edges are not 'random', but rather 'local')

Small-World Networks and Random Graphs

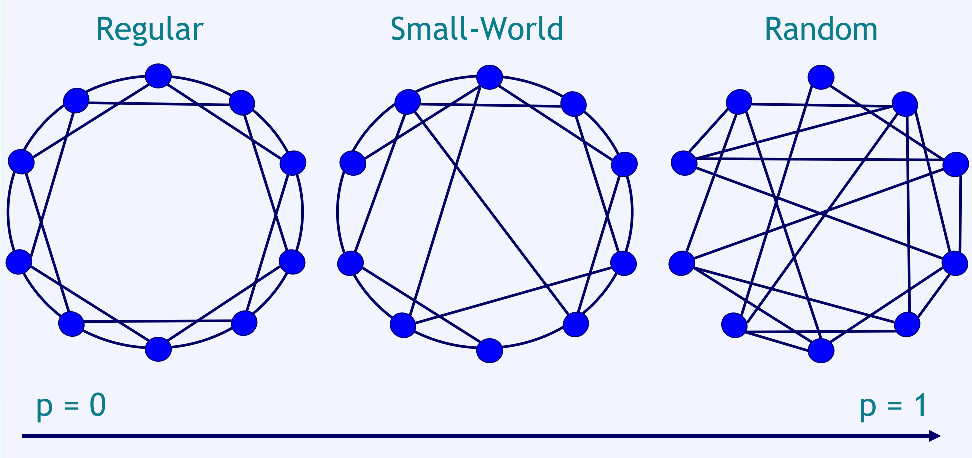
- Results
  - Compared to a random graph with same number of nodes
  - Diameters similar, slightly higher for real graph
  - Clustering coefficient orders of magnitude higher
- Definition of small-worlds network
  - Dense local clustering structure and small diameter comparable to that of a same-sized random graph

	$D_\oslash$(real)	$D_\oslash$(random)	C(real)	C(random)
Film collaboration	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.08	0.005
C. elegans	2.65	2.25	0.28	0.05

Constructing Small-World Graphs

- Put all *n* nodes on a ring, number them consecutively from 1 to *n*
- Connect each node with its *k* clockwise neighbors
- Traverse ring in clockwise order
- For every edge
  - Draw random number *r*
  - If *r* < *p*, then re-wire edge by selecting a random target node from the set of all nodes (no duplicates)
  - Otherwise keep old edge
- Different values of *p* give different graphs
  - If *p* is close to 0, then original structure mostly preserved
  - If *p* is close to 1, then new graph is random
  - Interesting things happen when *p* is somewhere in-between

Regular, Small-World, Random



Problems with Small-World Graphs

Small-world graphs explain why:

- Highly clustered graphs can have short average path lengths ("short cuts")

Small-world graphs do *NOT* explain why:

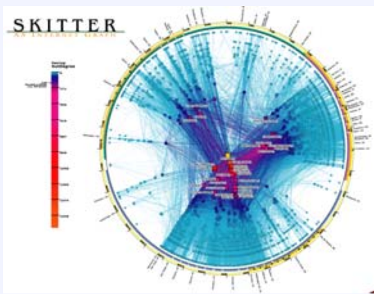
- This property emerges in real networks
  - Real networks are practically never ring-like

Further problem with small-world graphs:

- Nearly all nodes have same degree
- Not true for random graphs
- What about real networks?

Internet

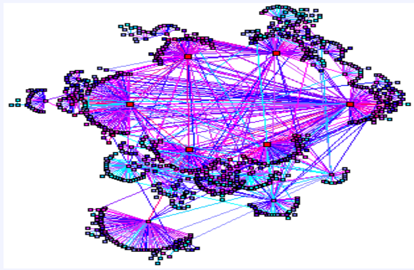
- Faloutsos et al. study from 99: Internet topology examined in 1998
  - AS-level topology, during 1998 Internet grew 45%

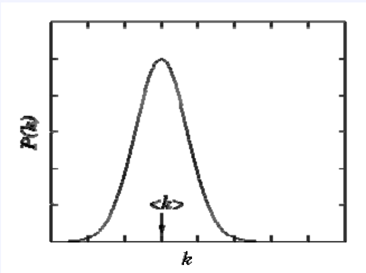


- Motivation:
  - What does the Internet look like?
  - Are there any topological properties that don't change over time?
  - How to gemerate Internet-like graphs for simulations?
- 4 key properties found,
  each follows a power-law;
  Sort nodes according to their (out)degree
  1. *Outdegree of a node is proportional to its rank to the power of a constant*
  2. *Number of nodes with same outdegree is proportional to the outdegree to the power of a constant*
  3. *Eigenvalues of a graph are proportional to the order to the power of a constant*
  4. *Total number of pairs of nodes within a distance d is proportional to d to the power of a constant*

World Wide Web

- Links between documents in the World Wide Web
  - 800 Mio. documents investigated (S. Lawrence, 1999)
- What was expected so far?
  - Number of links per web page: ⟨k⟩ ~ 6
  - Number of pages in the WWW: N_{WWW} ~ 10^9

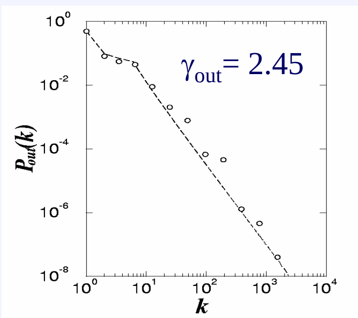




- Probability "page has 500 links": P(k=500) ~ 10^{-99}
- Number of pages with 500 links: N(k=500) ~ 10^{-90}

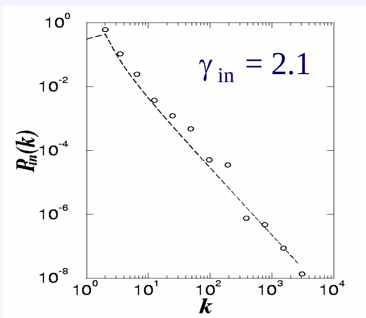
WWW: result of investigation

P(page has k links)



$P_{out}(k) \sim k^{-\gamma\ out}$

P(k pages link to this page)



$P_{in}(k) \sim k^{-\gamma\ in}$

P(k=500) ~ 10^{-6} N_{WWW} ~ 10^9 → N(k=500) ~ 10^3

Power Law Networks

- Also known as scale-free networks
- "Power Law" relationship for Web pages
  - The probability P(k) that a page has k links (or k other pages link to this page) is proportional to the number of links k to the power of y
- General "Power Law" Relationships
  - A certain characteristic k is - independent of the growth of the system - always proportional to k^a, whereby a is a constant (often $-2 < a < -4$)
- Power laws very common ("natural")
  - and power law networks exhibit small-world-effect
  - E.g. WWW: 19 degrees of separation
    (R. Albert et al, Nature (99); S. Lawrence et al, Nature (99))

Examples for Power Law Networks

- Economics
  - Pareto: income distribution
    (common simplification: 20% of population own 80% of the wealth)
  - Standardized price returns on individual stocks or stock indices
  - Sizes of companies and cities (Zipf's law)

> Pareto, Zipf distributions are the same (converted)

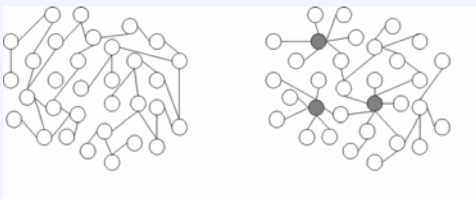
- Human networks
  - professional (e.g. collaborations between actors, scientists)
  - social (friendship, acquaintances)
  - Sexual-contact networks
- Many other natural occurrences
  - Distribution of English words (Zipf's law again)
  - Areas burnt in forest fires
  - Meteor impacts on the moon
- Internet also follows some power laws
  - Popularity of Web pages (possibly related to Zipf's law for English words?)
  - Connectivity of routers and Autonomous Systems
  - Gnutella's topology!

Barabasi-Albert-Model

- How do power law networks emerge?
  - In a network where new vertices (nodes) are added and new nodes tend to connect to well-connected nodes, the vertex connectivities follow a power-law
- Barabasi-Albert-Model: power-law network is constructed with two rules
  1. Network grows in time
  2. New node has preferences to whom it wants to connect
- Preferential connectivity modeled as
  - Each new node wants to connect to *m* other nodes
  - Probability that an existing node *j* gets one of the *m* connections is proportional to its degree *d(j)*
- New nodes tend to connect to well-connected nodes
- Another way of saying this: "the rich get richer"

Copying model

- Alternative generative model *(R. Kumar, P. Raghavan, et al. 2000)*
  - In each time step randomly copy one of the existing nodes keeping all its links
  - Connect the original node and the copy
  - Then randomly remove edges from both nodes with a very small probability, and for each removed edge randomly draw new target nodes
- In this model the probability of node *v* getting a new edge in some time step is proportional to its degree at that time
  - The more edges it has, the more likely it is that one of its neighbors is chosen for copying in the next time step
- In contrast to random networks, scale-free networks show a small number of well-connected hubs and many nodes with few connections



(a) Random network (b) Scale-free network

Robustness of Scale Free Networks

- Experiment: take network of 10000 nodes (random and power-law) and remove nodes randomly

- Random graph:
  - Take out 5% of nodes: Biggest component 9000 nodes
  - Take out 18% of nodes: No biggest component, all components between 1 and 100 nodes
  - Take out 45% of nodes: Only groups of 1 or 2 survive

- Power-law graph:
  - Take out 5% of nodes: Only isolated nodes break off
  - Take out 18% of nodes: Biggest component 8000 nodes
  - Take out 45% of nodes: Large cluster persists, fragments small

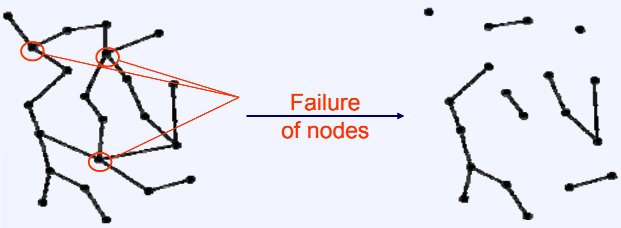
- Networks with power law exponent < 3 are very robust against random node failures
  - ONLY true for random failures!

Robustness of Scale-Free Networks /2

- Robustness against random failures = important property of networks with scale-free degree distribution
  - Remove a randomly chosen vertex v from a scale-free network: with high probability, it will be a low-degree vertex and thus the damage to the network will not be high

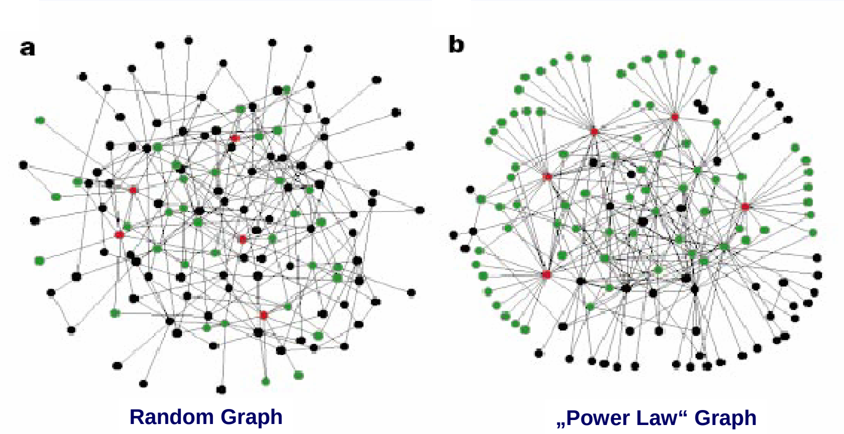
- But scale-free networks are very sensitive against attacks
  - If a malicious attacker removes the highest degree vertices first, the network will quickly decompose in very small components

- Note: random graphs are not robust against random failures, but not sensitive against attacks either (because all vertices more or less have the same degree)



Robustness of Scale-Free Networks /3

- Random failures vs. directed attacks



Kleinberg’s Small-World Navigability Model

- Small-world model and power law explain why short paths exist
- Missing piece in the puzzle: why can we find these paths?
  - Each node has only local information
  - Even if a short cut exists, how do people know about it?
  - Milgram’s experiment:
    - Some additional information (profession, address, hobbies etc.) is used to decide which neighbor is "closest" to recipient
    - results showed that first steps were the largest
- Kleinberg’s Small-World Model
  - Set of points in an *n* x *n* grid
  - Distance is the number of "steps" separating points
    - $d(i, j) = |x_i - x_j| + |y_i - y_j|$
- Construct graph as follows:
  - Every node *i* is connected to node *j* within distance *q*
  - For every node *i*, additional *q* edges are added. Probability that node *j* is selected is proportional to $d(i, j)^{-r}$, for some constant *r*

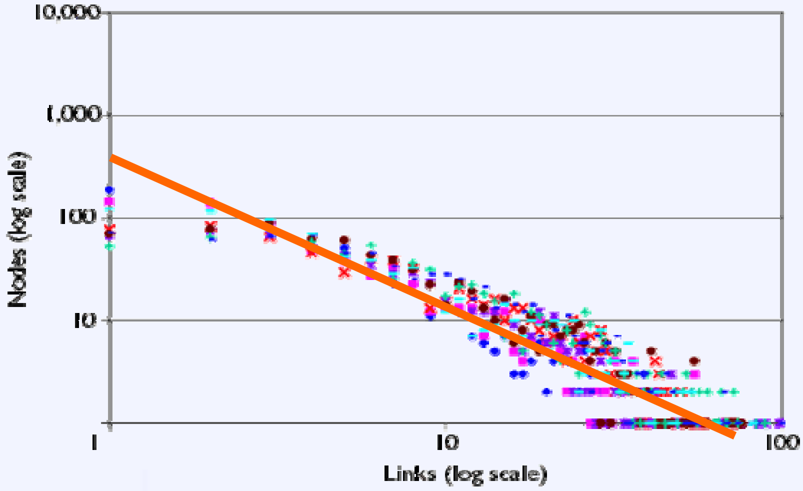
Navigation in Kleinberg's Model

- Simple greedy routing
  - If r=2, expected lookup time is $O(\log^2 n)$
  - If r≠2, expected lookup time is $\Omega(n^\varepsilon)$, ε depends on r
- Decentralized algorithm: sending node only knows its local neighbors, position of the target node on the grid, locations and long-range contacts of all nodes who come in contact of the message
- Can be shown: Number of messages needed is proportional to *O(log n)* *iff* *r=s* (s = number of dimensions)
  - Idea behind proof: for any r < s there are too few random edges to make paths short
  - For r > s there are too many random edges ⇒ too many choices for passing message
    - The message will make a (long) random walk through the network
- Kleinberg small worlds thus provide a way of building a peer-to-peer overlay network, in which a very simple, greedy and local routing protocol is applicable
  - Practical algorithm: Forward message to contact who is closest to target
  - Assumes some way of associating nodes with points in grid (know about "closest")
  - Compare with CAN DHT (later)

Unstructured P2P Networks

- What do real (unstructured) Peer-to-Peer Networks look like?
- Depends on the protocols used
  - It has been found that some peer-to-peer networks, e.g., Freenet, evolve voluntarily in a small-world with a high clustering coefficient and a small diameter
  - Analogously, some protocols, e.g., Gnutella, will implicitly generate a scale-free degree distribution
- Case study: Gnutella network
- How does the Gnutella network evolve?
  - Nodes with high degree answer more likely to Ping messages
  - Thus, they are more likely chosen as neighbor
  - Host caches always/often provide addresses of well connected nodes

Gnutella



Node degrees in Gnutella follow Power-Law rule

Gnutella /2

- Network diameter stayed nearly constant, though the network grew by one order of magnitude

- Robustness
  - Remember: we said that networks with power law exponent < 3 are very robust against random node failures
  - In Gnutella's case, the exponent is 2.3

- Theoretical experiment
  - Subset of Gnutella with 1771 nodes
  - Take out random 30% of nodes, network still survives
  - Take out 4% of best connected nodes, network splinters

- For more information on Gnutella, see:
  - Matei Ripeanu, Adriana Iamnitchi, Ian Foster: Mapping the Gnutella Network, IEEE Internet Computing, Jan/Feb 2002
  - Zeinalipour-Yazti, Folias, Faloutsos, "A Quantitative Analysis of the Gnutella Network Traffic", Tech. Rep. May 2002

Summary

- The network structure of a peer-to-peer system influences:
  - average necessary number of hops (path length)
  - possibility of greedy, decentralized routing algorithms
  - stability against random failures
  - sensitivity against attacks
  - redundancy of routing table entries (edges)
  - many other properties of the system build onto this network
- Important measures of a network structure are:
  - average path length
  - clustering coefficient
  - the degree distribution
- Next: how to build systems based on edge generation rules such that a network structure arises supporting the desired properties of the system

References / acknowledgments

- Slides from:
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